



Aalborg Universitet

AALBORG UNIVERSITY  
DENMARK

## Modelling of Crack Growth Processes by Introduction of a Characteristic Length

Brincker, Rune; Krenk, S.; Kirkegaard, Poul Henning; Rytter, A.

*Published in:*

Proc. Scandinavian Forum for Stochastic Mechanics, Aalborg, November 1991

*Publication date:*

1992

*Document Version*

Publisher's PDF, also known as Version of record

[Link to publication from Aalborg University](#)

*Citation for published version (APA):*

Brincker, R., Krenk, S., Kirkegaard, P. H., & Rytter, A. (1992). Modelling of Crack Growth Processes by Introduction of a Characteristic Length. In Aakjær, K. (ed.) (Ed.), *Proc. Scandinavian Forum for Stochastic Mechanics, Aalborg, November 1991* (pp. 1-14). Dept. of Building Technology and Structural Engineering, Aalborg University.

### General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal -

### Take down policy

If you believe that this document breaches copyright please contact us at [vbn@aub.aau.dk](mailto:vbn@aub.aau.dk) providing details, and we will remove access to the work immediately and investigate your claim.



Aalborg Universitet

AALBORG UNIVERSITY  
DENMARK

## Modelling of Crack Growth Processes by Introduction of a Characteristic Length

Brincker, Rune; Krenk, S.; Kirkegaard, Poul Henning; Rytter, A.

*Published in:*

Proc. Scandinavian Forum for Stochastic Mechanics, Aalborg, November 1991

*Publication date:*

1992

[Link to publication from Aalborg University](#)

*Citation for published version (APA):*

Brincker, R., Krenk, S., Kirkegaard, P. H., & Rytter, A. (1992). Modelling of Crack Growth Processes by Introduction of a Characteristic Length. In Aakjær, K. (ed.) (Ed.), Proc. Scandinavian Forum for Stochastic Mechanics, Aalborg, November 1991. (pp. 1-14). Dept. of Building Technology and Structural Engineering, Aalborg University.

### General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

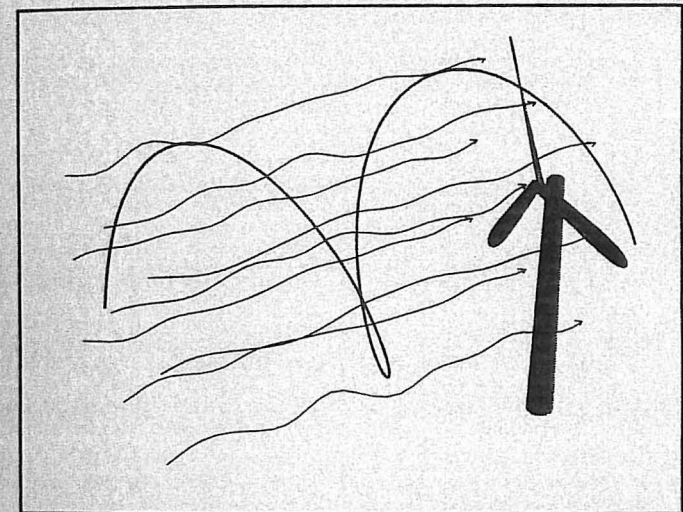
- ? Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- ? You may not further distribute the material or use it for any profit-making activity or commercial gain
- ? You may freely distribute the URL identifying the publication in the public portal ?

### Take down policy

If you believe that this document breaches copyright please contact us at [vbn@aub.aau.dk](mailto:vbn@aub.aau.dk) providing details, and we will remove access to the work immediately and investigate your claim.

# **SCANDINAVIAN FORUM FOR STOCHASTIC MECHANICS**

**UNIVERSITY OF AALBORG, DENMARK  
NOVEMBER 21-22, 1991**



**Department of Building Technology and Structural Engineering  
University of Aalborg, Aalborg, Denmark  
February 1992**

**ISSN 0902-7513 R9204**

Numerisk och experimentell kontroll av ny livslängdsmodell utgående från nivåöverskridande . . . . .	135
<i>T. Svensson, M. Holmgren</i>	
Methods for Estimation of Extreme Response of Flexible Risers . . . . .	143
<i>N. Sødahl, C. M. Larsen</i>	
The Structure of Atmospheric Turbulence at High Wind Speeds . . . . .	157
<i>J. M. Sørensen</i>	
List of participants . . . . .	169

## MODELLING OF CRACK GROWTH PROCESSES BY INTRODUCTION OF A CHARACTERISTIC LENGTH

R. Brincker, L. Gansted & J. P. Ulfkjær  
University of Aalborg  
Sohngaardsholmsvej 57, 9000 Aalborg, Denmark

### Abstract

It is illustrated how complex crack growth problems can be modelled in a simple way by introduction of a single additional length-scale parameter. The principle is illustrated on slow crack growth in viscoelastic materials, on stochastic modelling of fatigue crack growth, and on modelling of the stochastic fracture of concrete.

### 1. Introduction

Usually fracture mechanical models are based on linear elasticity and assumptions like straight cracks and homogenous and isotropic material. Even though some models might take local yielding and anisotropic effects into account, there is a large gap between the model world and reality where the cracks might be very irregular (see figure 1), the material might be inhomogenous with grains, inclusions, micro-cracks and initial stresses, and where the material response might be non-linear and time dependent.

It seems like a formidable task to try to model such effects. Fortunately it turns out, that some of the complicated relationships between the complex material micro-structure and the overall response is modelled quite well by simple models. It is illustrated, that by introduction of an extra material parameter into a crack theory, quite complicated phenomena can be modelled reasonably well. Three examples are given, slow crack growth in a viscoelastic material, fatigue crack growth in metals, and finally modelling of the stochastic fracture of concrete.

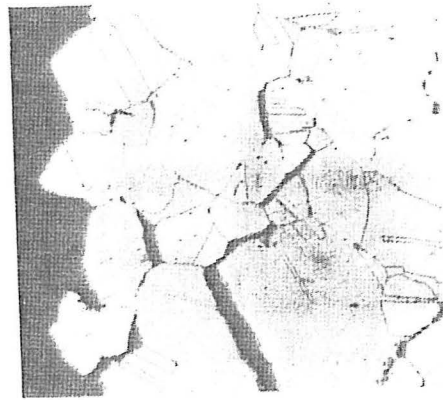


Figure 1 Intergranular cracking in a copper specimen (after Berenbaum et al [1]).

## 2. Slow Crack Growth in Viscoelastic Materials

It is well known that for materials like wood, concrete and plastics, the fracture process might be time-dependent. For instance, it might be observed that if a constant load is applied to a specimen, the specimen might only be able to resist the applied load a certain time. Much experimental work has been done in this area. One of the best known works is probably the investigations for determination of the long term strength of wood, Wood [2]. However slow crack extension under constant load might also be observed directly, Mueller et al [3].

Phenomena like reduced long term strength and slow crack growth under constant external loads might be governed by viscoelastic effects or by stress corrosion (environmental effects). However, only viscoelastic effects will be assumed to be present. Furthermore, it is assumed that the state of stress around the crack tip is approximately plane stress, pure mode I, and that the viscoelastic response can be modelled by linear viscoelasticity using a single creep function  $J(t)$  only (constant Poisson ratio).

In this case, it can be shown, Brincker [4] that the local crack tip fields can be described by two mode I crack tip parameters, the stress intensity factor  $K_I$  well known from elastic fracture mechanics, Hellan [5] and a deformation factor  $D_I$ . It can be shown, that if the loads are prescribed (pure stress boundary value problem) and constant in time,

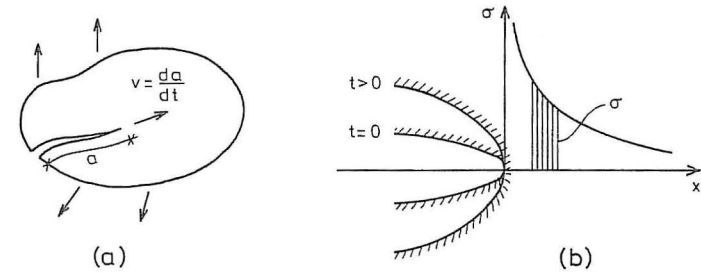


Figure 2 Slow crack growth in a linearly viscoelastic material

then the solution for a stationary crack (crack extension speed  $v = 0$ ) is

$$\sigma = \frac{K_I}{\sqrt{2\pi x}}; \quad u = 4D_I(t) \sqrt{\frac{-x}{2\pi}} \quad (1)$$

where  $\sigma$  is the normal stresses in the crack plane,  $u$  is the crack opening, and  $x$  is a coordinate measuring the distance from the crack tip, see figure 2.b. The stress intensity factor  $K_I$  is simply the same as for the corresponding elastic problem, and the deformation factor  $D_I(t)$  is given by

$$D_I(t) = K_I J(t) \quad (2)$$

where  $J(t)$  is the creep function corresponding to  $1/E$  where  $E$  is Youngs modulus. If the crack has been stationary since the loading of the specimen, the time  $t$  is simply the time since the application of the loading, and if the crack has been extending in an earlier stage, then  $t$  is the time since the crack became stationary.

If the crack is extending (crack extension speed  $v > 0$ ) then the corresponding solution is

$$\sigma = \frac{K_I}{\sqrt{2\pi x}}; \quad u = 4D_I(0+) \sqrt{\frac{-x}{2\pi}} \quad (3)$$

where  $D_I(0+)$  is the initial value of the deformation factor  $D_I(0+) = K_I/E$ .



In classical fracture mechanics the crack extension criterion is given by

$$\mathcal{G} = \mathcal{G}_c \quad (4)$$

where the state parameter  $\mathcal{G}$  is defined as the derivative of the total potential energy  $\Pi$  with respect to the crack length  $\mathcal{G} = -\partial\Pi/\partial a = K_I^2/E$ , and where  $\mathcal{G}_c$  is a strength parameter associated with the energy needed to create a new surface by a fracture process. In linearly viscoelasticity no potential energy is defined, and therefore, a definition of a corresponding state parameter is not straight forward. If however, the energy needed to form a new surface by a fracture process is still taken as a material property, then, by assuming isothermal conditions and using the principle of energy conservation, a corresponding crack extension criteria can be derived, Brincker [6]

$$\mathcal{G}(t) = \mathcal{G}_c; \quad \mathcal{G}(t) = K_I(2D_I(t) - D_I(2t)) \quad (5)$$

The solutions (1), (2) and the crack extension criterion given by (5), form the simplest possible basis for a model of slow crack propagation in viscoelastic materials. Furthermore the equations does not violate any basic equations like the equilibrium equations or the thermodynamical equations.

Unfortunately, the solutions does not form an applicable basis for modelling slow crack growth. A model based on these equations only will simply not work. Assuming that  $\mathcal{G}(0+) < \mathcal{G}_c$ , i.e. the crack tip is stationary and assuming that after some time  $\Delta t$  the crack has opened so much that the crack extension criterion is satisfied  $\mathcal{G}(\Delta t) = \mathcal{G}_c$ , the crack will start to run. According to eq. (3) however, the deformation factor  $D_I$  is reduced to its initial value at the very moment the crack starts to grow. Therefore, one have a dilemma: as soon as the crack extension criterion is satisfied and the crack starts to grow, the crack tip deformation field is changed, and the crack extension criterion is no longer satisfied. The derived equations do neither allow the crack to grow nor to be stationary.

The reason is that the model is to simple. The equations are in agreement with the basic continuum mechanical equations, but do not form a sufficient basis for modelling the crack growth process. The problem can be solved in different ways.

One way is to use a non-linear theory. If a suitable non-linear theory is used, the local stress field will be non-singular, the non-smooth transition from stationary displacement field to the displacement field of a running crack will vanish, and the dilemma will be gone. It is extremely more difficult however to model the problem by non-linear fracture mechanical models in the viscoelastic case. It has been tried by many researchers, and different models have been proposed, Shapery [7], McCartney [8], Nielsen [9]. The problem with these models is that the description of the non-linear viscoelasticity is uncertain, and that no simple crack extension criteria in agreement with basic thermodynamical laws can be formulated.

A much simpler approach is just to add one more material parameter to the above established formulas. One could simply assume that the crack extends in steps with the

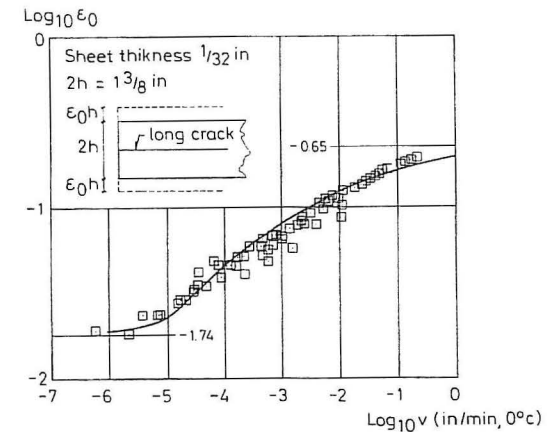


Figure 3 Experimental and theoretical results for slow crack growth in the infinite strip. Experimental results by Mueller et al [3]. Analytical results after Brincker [22]

length  $\Delta a$ . It could be looked upon as a simple way of modelling the fact that materials are not continua, but are discretized into building stones at some level. At some level the material behaviour is not governed by average properties, but by local properties of grains, micro-cracks, fibers or atoms. It might therefore be reasonable to assume, that if the crack tip moves, it moves at least the size of one typical building stone. This step length is modelled by introduction of the length parameter  $\Delta a$ .

Once this parameter is defined, a simple and complete model is formed. At each stage of the fracture process, the time  $\Delta t$  spent in each position is obtained by equation (1) and (5), and the velocity of the crack is simply given by

$$v = \frac{\Delta a}{\Delta t} \quad (6)$$

The model works quite well. In figure 3 theoretical results are compared with measurements on a rubber material made by Mueller et al [3]. The creep function was also measured by Mueller. The model was only calibrated by fitting the parameter  $\Delta a$  which was found to  $\Delta a = 30\text{\AA}$ , Brincker [22].

### 3. Stochastic modelling of fatigue crack growth

The idea introduced in the preceding section of using a single crack length parameter to model the complex micro-level mechanics of a material, is now used to formulate a simple stochastic model of fatigue crack growth.

The length parameter is introduced by assuming that the crack growth process is a discrete Markov process with the states

$$a_j = a_0 + j\Delta a; \quad j = 0, 1, 2, \dots \quad (7)$$

where  $a$  is the crack length. If viscoelastic effects are excluded, the material itself does not have any memory, and therefore in principle the Markov assumption is valid. One can discuss of cause which parameters should be included in the state vector. If one would like to describe more complicated phenomena like acceleration and deceleration, some additional state variables like for instance the closure stress or the effective stress intensity factor width could be included, Schijve [10]. In this case however, only the simple case of constant amplitude loading will be considered.

The crack is assumed only to move one step at a time. Therefore, in a given position there is only two possibilities each time a load cycle is applied:

$$\begin{aligned} \mathcal{A} &: \text{the crack propagates } \Delta a \\ \mathcal{B} &: \text{the crack does not propagate} \end{aligned} \quad (8)$$

In each state therefore one have just a simple coin tossing problem completely described by the transition probability  $P\{\mathcal{A}\} = q$ . The number of load cycles  $\Delta N$  the crack can resist before it propagates is a stochastic variable known to follow a geometrical distribution, Papoulis [11]

$$E[\Delta N] = \frac{1}{q} \quad \text{and} \quad \text{Var}[\Delta N] = \frac{1-q}{q^2} \quad (9)$$

One can imagine that the stochastic variable  $\Delta N$  is generated by an unknown stochastic system  $\Delta N = S(a, \text{loading, geometry, etc})$ . However some information on the expectation of the variable is known. Using the Paris equation

$$\frac{da}{dN} = C\Delta K^m \quad (10)$$

where  $\Delta K$  is the stress intensity factor range and  $C$  and  $m$  are the Paris constants, a simple equation is obtained for the expectation of the stochastic variable  $\Delta N$

$$E[\Delta N] = \frac{\Delta a}{C\Delta K^m} \quad (11)$$

or from (9) the transition probability is found to

$$q = \frac{C\Delta K^m}{\Delta a} \quad (12)$$

The above derivation can be looked upon as a simple way of estimating the transition probabilities in a Bogdanoff model, Bogdanoff and Kozin [12]. The advantage of having a model, as described above, instead of a usual Bogdanoff model is, that instead of estimating the transition probabilities in each crack position as independent variables, only one parameter has to be estimated. Furthermore, the influence from the geometry and the loading conditions is directly established, and therefore, results can easily be transferred between different specimens. This is not the case for usual Bogdanoff models.

Now let the state  $j = c$  denote the final critical state where the static crack extension criterion is satisfied, see eq. (4), and let  $N$  denote the total number of cycles to reach the final state

$$N = \sum_{j=0}^c \Delta N_j \quad (13)$$

It is easily verified that taking the expectation of  $N$  and replacing the sum by an integral, the same result is obtained as when integrating Paris' equation directly. Besides an analytical result for the variance can be derived by the model. Assuming that  $q \ll 1$ , and replacing the sum by an integral, the following general expression for the variance can be derived, Gansted et al. [13]

$$\begin{aligned} \text{Var}[N] &= \sum_{j=0}^c \frac{1-q_j}{q_j^2} \\ &\cong \frac{\Delta a}{C^2} \int_{a_0}^{a_c} \Delta K(a)^{-2m} da \end{aligned} \quad (14)$$

For instance for the infinite sheet where  $\Delta K = \Delta\sigma\sqrt{\pi a}$  the following simple result is obtained

$$\text{Var}[N] = \frac{a_0}{\gamma_0^2} \frac{\Delta a}{m-1} \quad (15)$$

where  $\gamma_0$  is the initial expected crack growth rate  $\gamma_0 = da/dN$  calculated by Paris' equation for  $a = a_0$ .

In figures 4 and 5 theoretical and experimental results for a mild steel are compared, Gansted [14]. The theoretical model is calibrated by fitting  $\Delta a$  to give the right final variance. The value was found to  $\Delta a = 0.055$  mm. It is seen that the average propagation rate is underestimated by the model in the final stage where the specimen is close to static fracture. This is believed not to have anything to do with the basic ideas of the model presented here, but is probably due to the fact that the Paris equation is underestimating the crack growth rate close to static fracture. Note that the model concept allows the use of any crack growth equation. For instance, other well known

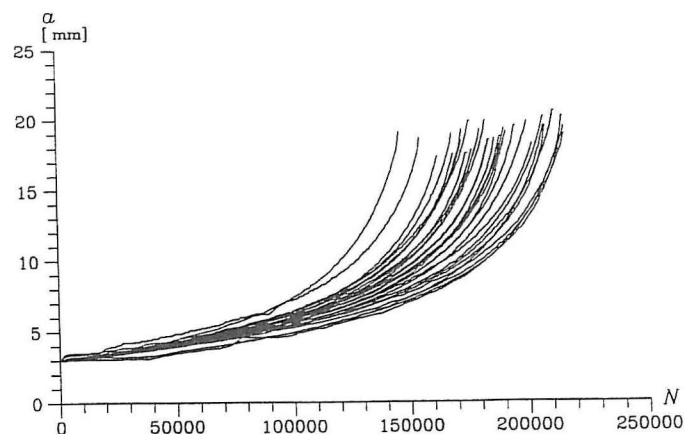


Figure 4 Crack length as a function of number of load cycles. Experimental results after Gansted [14]

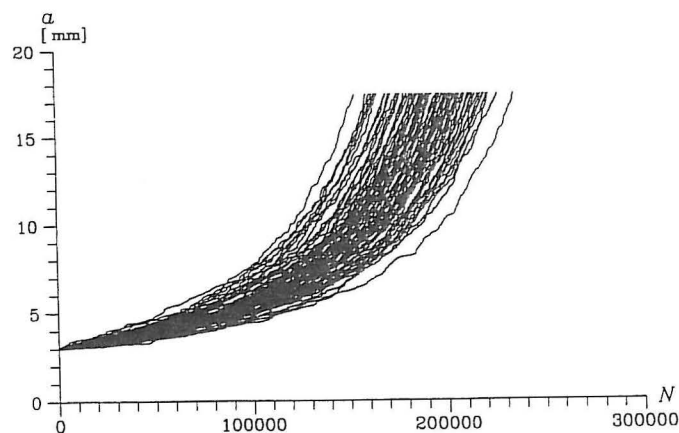


Figure 5 Crack length as a function of number of load cycles. Results simulated by the model. After Gansted [14]

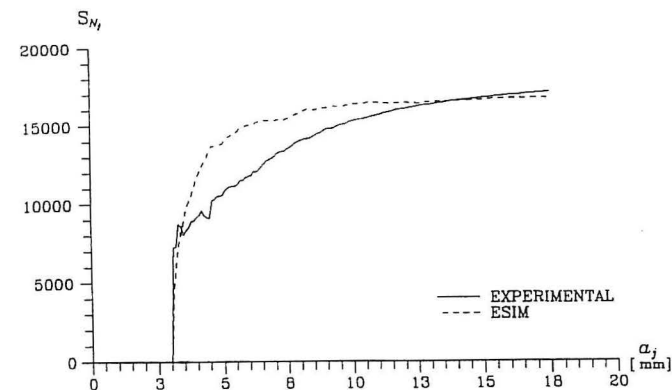


Figure 6 Simulated and experimental results for the variance of the number of cycles required to propagate the crack to a certain length  $a$  as a function of the crack length  $a$ . After Gansted [14]

crack growth equations like the Forman equation, Gurney [15], could have been used to take account of fracture toughness effects.

In figure 6 is shown the theoretical and experimental variance as a function of the crack length. The theoretical variance was found by simulation. It is seen that the presented model give to much variance in the beginning. This is only natural since to much of the variance is modelled by local disturbances and that no spatial correlation is taken into account. It must be kept in mind however that the model is extremely simple. Note for instance that to make Paris' law stochastic would require 3 addition parameters in the general case. A better modelling could be archived by allowing the crack to jump more than step at a time. One would expect that this would give a more slowly development of the variance.

#### 4. Modelling of the stochastic fracture of concrete

During the last 10-15 years the fracture mechanics of inhomogeneous and quasi-brittle materials like concrete and rock has been intensively studied, and new analysis tools have been developed, [16], [17].

One of the most well established models for concrete fracture is the Hillerborg model where the behaviour of the material in the fracture zone is modelled by a stress-crack-



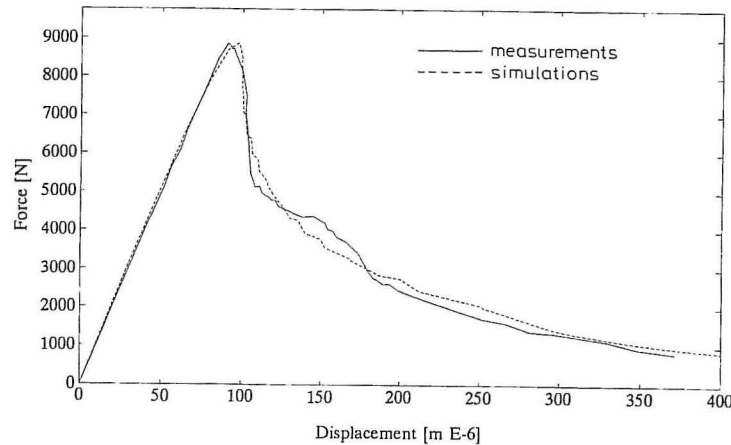


Figure 7 Typical result comparing experimental force-displacement curves with force-displacement curves simulated by a Hillerborg model with bi-linear stress-crack-opening relation

opening ( $\sigma - w$ ) relation and the material outside the fracture zone (the bulk response) is modelled by linear elasticity. Quite good results can be obtained by using simple  $\sigma - w$  relations like a bi-linear relation. Figure 7 show a typical result for description of the load-displacement ( $F - u$ ) relation for an un-reinforced concrete beam. The beam was tested as a part of a larger experimental investigation of size effects in un-reinforced concrete beams, UHkjær and Brincker [18], and the theoretical results were obtained by simulation using the so-called direct substructure method, Brincker & Dahl [19].

The figure illustrates a typical observation, namely, that the average response seems to be well described by the Hillerborg model, and that a large part of the deviation from the response predicted by the model seems to be random. It would therefore be interesting to be able to model the random effects and by such a model be able to predict for instance how the variance of the ultimate load depends on the beam size. Only a few attempts have been made to formulate stochastic models for concrete fracture although, some more complicated models like models based on Stochastic Finite elements have been formulated, Elices & Planas [20].

In this section some preliminary results will be given illustrating the possibilities of modifying the Hillerborg model to incorporate stochastic response.

For normal size beams, the size of the fracture zone, the size of the maximum aggregate, and the depth of the beam are all comparable quantities, the maximum aggregate size being somewhat smaller than the two others. This implies, that a simple approach like in the two preceding sections is not sufficient - a more detailed description will be needed.

A simulation model is established by assuming the fracture energy  $\mathcal{G}_F$  to be a 2D random field over the beam cross section. The fracture energy is the area under the  $\sigma - w$  relation. It is assumed that the shape of the  $\sigma - w$  relation is constant, and that the stresses for a given crack opening displacement are proportional to the fracture energy. This implies, that the fracture energy and the tensile strength are completely correlated.

The fracture energy field is assumed to be described as white noise filtered through a 2D isotropic, symmetrical 2nd-order filter

$$\mathcal{G}_F(x, y) = \mathcal{G}_{F0} + h^s(x) * h^s(y) * e(x, y) \quad (16)$$

where  $x, y$  are the cross section coordinates,  $\mathcal{G}_{F0}$  is the mean value of the fracture energy,  $*$  denotes convolution,  $e(\cdot, \cdot)$  is a zero mean Gaussian white noise field with zero variance outside the cross-section, and  $h^s(\cdot)$ ,  $h^s(\cdot)$  are the symmetrical impulse response functions for a second order system with one degree of freedom. In time domain formulations distinction is made between the future and the past, and the normal non-symmetrical (causal) response function  $h(t)$  is used. In a spatial formulation no such distinction can be made, and therefore, the corresponding symmetrical response function  $h^s(t) = (h(t) + h(-t))/2$  is used.

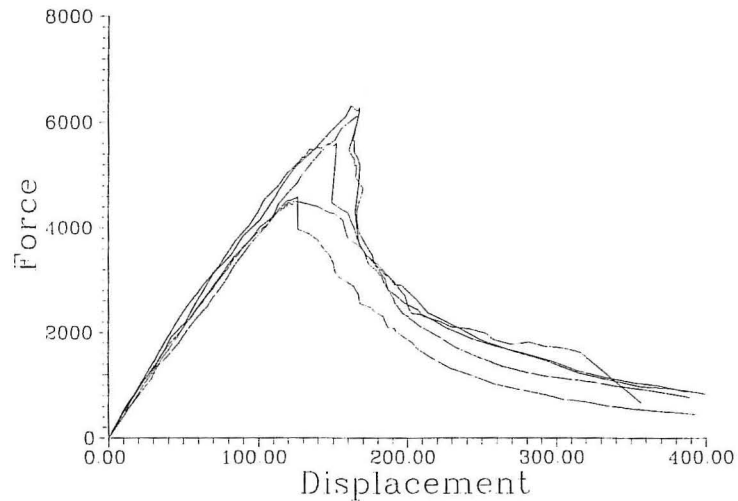
The 2D filter is defined by its spatial eigenfrequency and damping ratio. The damping ratio is chosen as  $\zeta = 1$ , and the spatial eigenfrequency is taken as  $1/\rho$ , where  $\rho$  is a characteristic length describing the spatial correlation. In practice a discrete model must be used to match the discrete direct substructure model. In the substructure method a rather coarse discretization is often used with only 10 or 20 nodes over the cross section. Therefore, care must be taken in order not to let the coarse discretization have a direct influence on the statistical properties of the simulated fracture energy field. This can be done using a simulation method that preserves prescribed covariance properties irrespective of the discretization.

Covariance properties can be preserved using an ARMA (2,1) model. In a non-symmetrical (causal) formulation in 1D the fracture energy field  $\mathcal{G}_F(i\Delta x) = \mathcal{G}_{F0} + g_i$  where  $g_i$  is given by

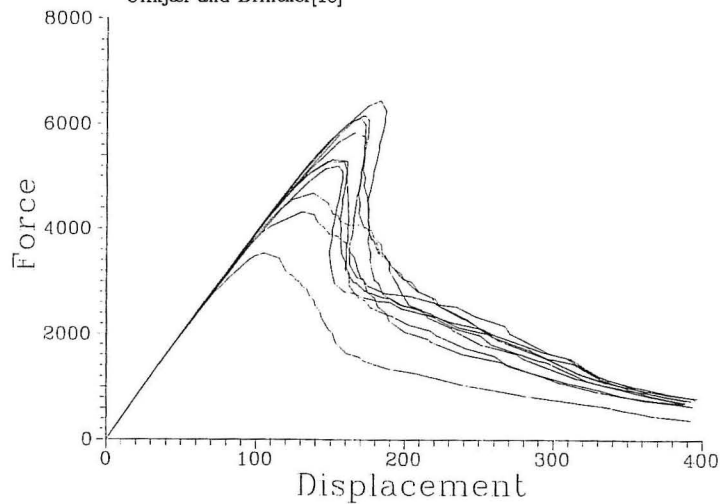
$$g_i = \Phi_1 g_{i-1} + \Phi_2 g_{i-2} + e_i - \Theta e_{i-1} \quad (17)$$

where the Auto Regressive parameters  $\Phi_1, \Phi_2$  and the Moving average parameter  $\Theta$  depends on the spatial frequency  $1/\rho$ , the damping ratio  $\zeta$  and the discretization step  $\Delta x$ . Analytical solutions might be found in Pandit & Wu [21]. In practice the field is simulated using a 2D symmetrical ARMA (2,1) model by applying the non-symmetrical simple 1D ARMA (2,1) model (17) from all four sides of a 2D white noise field. Afterwards the variance  $\lambda$  of the fracture energy field is adjusted to the prescribed value.

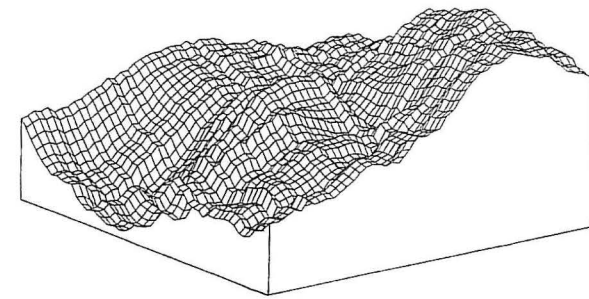
This model incorporates two parameters: the characteristic length  $\rho$  and the variance  $\lambda$  of the fracture energy field. Figure 10 shows a simulation of the fracture energy field for a square cross section with side length  $a$  and the relative characteristic length  $r = \rho/a = 0.25$ .



**Figure 8** Experimental load-displacement curves for a 100 × 100 mm beam section (length = 800 mm) with a maximum aggregate size of 8 mm. Experimental results after Ulfkjær and Brincker[18]



**Figure 9** Simulated load-displacement curves for a 100 × 100 mm beam section (length = 800 mm) with a characteristic length of 25 mm. For this case the variance  $\lambda$  of the fracture energy was chosen such that  $\sqrt{\lambda} = G_{f0}/2$



**Figure 10** Typical result for simulated fracture energy field for  $\zeta = 1$

Simulation results show a strong size effect on the variance on simulated load-displacement curves. If the "material parameters"  $\rho$  and  $\lambda$  are kept constant, a significant increase of the variance was observed when the specimen size was decreased. Some typical results are shown in figures 8 and 9.

No attempts have yet been made to calibrate the model or compare the model with experimental results. If the results of the model are in good agreement with experiments, then one could hope that the correlation length  $\rho$  is related to the maximum aggregate size and that the variance  $\lambda$  is related to the difference between mortar strength and aggregate strength.

## References

- [1] Berenbaum, L., J. Brett and L. Seigle. Final Report TR61-1031. Office of Naval Research, Washington, D.C., 1961.
- [2] Wood, L.W.: *Relation of Strength of Wood to duration of Load*. U.S. Forest Production Laboratory. Report No. R-1916, Madison Wis., 1951.
- [3] Mueller, H.K. and W.G. Knauss: *Crack Propagation in a Linearly Viscoelastic Strip*. Journal of Applied Mechanics, June 1971, pp. 483-488.
- [4] Brincker, R.: *Crack Tip Parameters for Growing Cracks in Linear Viscoelastic Materials*. The First International Conference on Computer Aided Assessment and Control of Localized Damage, 26 - 28 June 1990, Portsmouth, UK. Proc. Vol. 2, pp. 85-98.
- [5] Hellan, K.: *Introduction to Fracture Mechanics*. McGraw-Hill Book Compagny, 1985.

- [6] Brincker, R.: *Plane Revneudbredelsesproblemer i Lineært Viscoelastiske Materialer. Revnemodeller og udbredelseskriterier..* Afdelingen for Bærende Konstruktioner, Danmarks Tekniske Højskole, Rapport No. R-132, 1983.
- [7] Schapery, R.A.: *A Theory of Crack Initiation and Growth in Viscoelastic Media.* International Journal of Fracture. Part I: Vol. 11 No. 1, February 1975, pp. 141-158. Part II: Vol. 11, No. 3, June 1975, pp. 369-387. Part III: Vol. 11, No. 4, August 1975, pp. 549-562.
- [8] McCartney L.N.: *Crack Growth Laws for a Variety of Viscoelastic Solids Using Energy and COD Fracture Criteria.* International Journal of Fracture, vol. 19, No. 1, February 1979, pp. 31-40.
- [9] Nielsen, L.F.: *A Lifetime Analysis of Cracked Linear Viscoelastic Materials - with Special Reference to Wood.* Proc. of IUFRO 55.02 (Meeting of int. Union of Forestry Research Organizations Wood Engineering Group), Borås, Sweden, May 1982, pp. 151-178.
- [10] Schijve, J.: *Four Lectures on Fatigue Crack Growth.* Engineering Fracture Mechanics. Vol. 11, pp. 167-221, 1979.
- [11] Papoulis, A.: *Probability, Random Variables, and Stochastic Processes.* McGraw-Hill International Book Company, 1985.
- [12] Bogdanoff, J.L. and F. Kozin: *Probabilistic Models of Cumulative Damage.* John Wiley & Sons, 1985.
- [13] Gansted, L., R. Brincker and L.P. Hansen: *Fracture Mechanical Markov Chain Crack Growth Model.* Engineering Fracture Mechanics, Vol. 38, No. 6, 1991, pp. 475-489.
- [14] Gansted, L.: *Analysis and Description of High-Cycle Stochastic Fatigue in Steel.* Ph.D.-Thesis, report R9135, November 1991.
- [15] Gurney, T.R.: *Fatigue of Welded Structures.* Cambridge University Press, 2. ed. 1979.
- [16] Elfgrén, L. (editor): *Fracture Mechanics of Concrete Structures. From Theory to Applications.* RILEM Report, Chapman and Hall, 1989.
- [17] Elfgrén, L. and S.P. Shah (editors): *Analysis of Concrete Structures by Fracture Mechanics.* Proceedings of the International RILEM Workshop. Chapman and Hall, 1991.
- [18] Ulfkjær, J.P. and R. Brincker: *An Experimental Investigation of Size Effects in High-Strength Concrete Beams.* To be published.
- [19] Brincker, R. and H. Dahl: *On the Fictitious Crack Model of Concrete Fracture.* Magazine of Concrete Research, Vol. 41, No. 147, June 1989, pp. 79 - 86.
- [20] Elices, M. and J. Planas: *Material Models.* Chapter 3 in [14].
- [21] Pandit, S.M. and S.M. Wu: *Time Series Analysis with Applications.* John Wiley and Sons, 1983.
- [22] Brincker, R.: *Plane Revneudbredelsesproblemer i Lineært Viscoelastiske Materialer. Teori og anvendelser..* Afdelingen for Bærende Konstruktioner, Danmarks Tekniske Højskole, Rapport No. R-133, 1984.

# DETERMINATION OF AMPLIFICATION FACTOR FOR BENDING STRENGTH OF CONTINUOUS TIMBER BEAMS BY MONTE-CARLO SIMULATION

I. Czmocho  
Chalmers University of Technology  
S-412 96 Gothenburg, Sweden

Timber as a building material is a composition of clear wood and knots. Knots have a major influence on the mechanical properties of timber. The bending strength of cross-section with knots is considerably reduced due to:

- grain distortions in the vicinity of knots,
- stress concentrations caused by knot holes and encased knots,
- differences between the properties of the knot and the surrounding wood.

The fact, that the failure of timber beam subjected to bending almost always occurs in the vicinity of knot, is the basis of a simple model of lengthwise variation of strength of a structural timber. The main assumptions of weak-zones model are:

- timber is composed of short weak zones connected by sections of clear wood
- the weak zones (knots or group of knots) are randomly distributed
- failure occurs only in the middle of the weak zones
- the strengths of the weak zones are random.

Hence, the spatial distribution of bending strength of timber beam is modelled by means of the composite random point series: random series of bending strength is assigned to randomly distributed weak zones. It is assumed that both random series are stationary.

The bending strength of the timber beam is characterized by length and load configuration effects. The load configuration used for assigning characteristic strength values is four-points load with region of constant bending moment in the middle of the beam. According to the standard test procedure the weakest cross-section chosen by subjective judgement (e.g. the cross-section with the most unfavourable knot) should be placed in the region of constant bending moment. In that way the characteristic value of